Power Control and Resource Allocation for Multi-Cell OFDM Networks with Load Coupling

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In this paper, we study the power control and resource allocation problem in downlink orthogonal frequency division multiplexing networks where mutual interference exists among cells. This mutual relation is characterized by the load coupling model. Both cell load and transmit power, where cell load measures the average proportion of resource usage in the cell, interact via the coupling model. We consider three kinds of problems, sum power minimization, sum rate maximization and sum energy efficiency maximization. For each problem, we develop a correspondingly distributed power control and resource allocation algorithm with low complexity. Numerical results verify the effectiveness of our proposed algorithms compared with the existing algorithm.

***Index Terms*—Load coupling, power control, resource allocation, energy efficiency.**

I. Introduction

Sum power minimization [2]–[5], sum rate maximization [6]–[9] and sum energy efficiency maximization [10]–[14] are three fundamental optimization problems in wireless communication networks. To solve these three problems, power control and resource allocation are often considered [15]–[17].

For a multi-cell system where each subcarrier is taken by at most one user, [18] showed that Lagrange dual decomposition method can be used to find the optimal solution to sum power minimization problem with large number of subcarriers. Besides, for distributed orthogonal frequency division multiplexing (OFDM) femtocell networks, [19] introduced a simple self-organization rule, based on minimizing cell transmit power. In [20], resource allocation problem in OFDM-based cognitive radio networks was formulated as a mixed integer problem. This kind of resource allocation problems were also investigated in other aspects, such as relay networks [21] and wireless virtualization networks [22].

Different from the above power control and resource allocation problems in [18]–[22], where subcarrier assignment with integer variable is considered, the variables of resource allocation problems in [23]–[29] with load coupling model are all continuous. The load coupling model was first introduced in [30], where the load of a cell is defined as the average level of usage of time and frequency resources. Specifically, the high load value of a base station (BS) represents a high probability for other BSs to receive interference. This load coupling model has been shown to give a good approximation for a multi-cell network especially at high data arrival rates in [23]. Since the load coupling model has a good structure with high accuracy for characterizing inter-cell interference, it has been used in many applications for multi-cell networks [24], such as load balancing [25], [26], location planning [27], data offloading [28], and user association [29].

Previous works [23]–[30] with load coupling model all assumed fixed transmit power of BSs. To tackle the power control and resource allocation problem in multi-cell networks with load coupling, both load and power should be jointly optimized. The sum power minimization problem was considered in [31], where both load and power of each cell are incorporated into the signal-to-interference-noise-ratio (SINR) coupling model. The coupling was implicitly characterized with load and power as the variables of interest using non-linear load and power coupling equation. It was analytically shown that operating at full load is optimal, and an iterative power adjustment algorithm for all BSs was provided to achieve the full load. In [31], the transmit power is different for users in different cells, but the same for users in the same cell, even though power control for users in the same cell can further improve the system performance. Moreover, the above works [23]–[31] all assumed that the channel gains for different subcarriers are the same, which ignored the channel diversity gains among different subcarriers. With taking into account different channel gains on different subcarriers, one challenge is to allocate total power of each BS on each subcarrier while satisfying the maximal transmit power constraint. Moreover, after obtaining the total transmit power of each BS on each subcarrier, the other challenge is to allocate different power to different users according to different channel gains.

Comparing with the existing ones, there are two important advances in this paper. The first one is that we extend the load coupling model in [31] for multi-cell networks with frequency selective fading channels and unequal power allocation in each cell. The second one is that for sum power minimization or sum rate maximization or sum energy efficiency maximization problem, we show that it is a convex problem of each BS with fixed strategies of other BSs, which helps design the distributed algorithm.

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In this paper, we study the sum power minimization, sum rate maximization and sum energy efficiency maximization problems for multi-cell OFDM networks. We consider the load coupling model in frequency selective fading channels. Comparing with the existing works, the main contributions of this paper are summarized as follows:

- Differently from [31], where the channel gains between one user and the BS are the same for different subcarriers, the channel gains on different subcarriers in this paper are modeled as different values. Besides, we extend the load coupling model in [31] where the transmit power for users in the same cell is the same to the case that users in the same cell are allocated with different power.
- Different from our previous conference paper [1], where only sum power minimization problem was investigated, we formulate a unified framework for multi-cell OFDM networks to minimize sum power, maximize sum rate or maximize sum energy efficiency in this paper.
- We provide three low-complexity distributed algorithms to solve the corresponding sum power minimization problem, sum rate maximization problem and sum energy efficiency problem. Moreover, the complexity analysis and the implementation method are also provided.

This paper is organized as follows. In Section II, we introduce the system model and provide a unified formulation of three representative optimization problems. Sum power minimization problem, sum rate maximization problem and sum energy efficiency maximization problem are studied in Section III, IV and V, respectively. Some numerical results are displayed in Section VI and conclusions are finally drawn in Section VII.

II. System Model

Consider a multi-cell OFDM network consisting of N BSs denoted as the set $\mathcal{N} = \{1, 2, \cdots, N\}$, as shown in Fig. 1. Each BS $i \in \mathcal{N}$ serves one unique group of users, denoted by the set $\mathcal{J}_i = \{J_{i-1}+1, J_{i-1}+2, \cdots, J_i\}$, where $J_0 = 0$, $J_i = \sum_{l=1}^{i} |\mathcal{J}_l|$, $|\cdot|$ is the cardinality of a set and $|\mathcal{J}_i| \geq 1$. We focus on the downlink scenarios where each BS transmits data to different users with different power and time fractions on different subcarriers. Each BS is assumed to have R subcarriers, denoted by the set $\mathcal{R} = \{1, 2, \cdots, R\}$. On subcarrier $r \in \mathcal{R}$, BS i transmits with power p_{ij}^r to user $j \in \mathcal{J}_i$. For notational convenience, we collect all transmit power of BS i as vector $\boldsymbol{p}_i = [p_{i(J_{i-1}+1)}^1, \cdots, p_{iJ_i}^1, \cdots, p_{i(J_{i-1}+1)}^R, \cdots, p_{iJ_i}^R]$, and denote $\boldsymbol{p} = [\boldsymbol{p}_1, \cdots, \boldsymbol{p}_N]$.

Assume that each user can use all subcarriers, and users in the same cell cannot use the same subcarrier at the same time. To show this, we introduce load variable $m_{ij}^r \in [0, 1]$, which is regarded as the fraction of subcarrier r allocated to user $j \in \mathcal{J}_i$ by time division. Then, the load of BS i on subcarrier r can be calculated by the summation of load for serving every user $j \in \mathcal{J}_i$ on subcarrier r, i.e., $\sum_{j\in\mathcal{J}_i} m_{ij}^r$. To ensure that users in the same cell do not occupy the same subcarrier at the same time, we must have $\sum_{j\in\mathcal{J}_i} m_{ij}^r \leq 1$, $\forall r \in \mathcal{R}$. Denote $\boldsymbol{m}_i = [m_{i(J_{i-1}+1)}^1, \cdots, m_{iJ_i}^1, \cdots, m_{i(J_{i-1}+1)}^R, \cdots, m_{iJ_i}^R]$ and $\boldsymbol{m} = [\boldsymbol{m}_1, \cdots, \boldsymbol{m}_N]$.

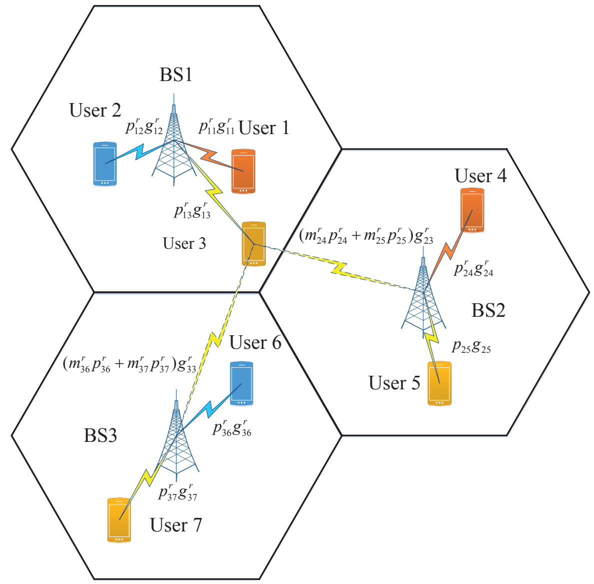

Fig. 1. System model.

We adopt the load and power coupling model for the multi-cell OFDM network. If the resource allocation is randomly distributed and we consider the long-term average interference from other BSs, the SINR model of user $j \in \mathcal{J}_i$ served by BS i on subcarrier r can be formulated as [30]–[33],

$$\eta_{ij}^r = \frac{p_{ij}^r g_{ij}^r}{\sum_{k\in\mathcal{N}\setminus\{i\}} \sum_{l\in\mathcal{J}_k} m_{kl}^r p_{kl}^r g_{kj}^r + \sigma^2}, \tag{1}$$

where g_{ij}^r is the channel gain from BS i to user j on subcarrier r and σ^2 represents the noise power. Intuitively, load proportion m_{kl}^r can be interpreted as the probability of receiving interference from BS k on subcarrier r for meeting the rate demand of user $l \in \mathcal{J}_k$. Thus, the combined term $m_{kl}^r p_{kl}^r g_{kj}^r \in [0, p_{kl}^r g_{kj}^r]$ is interpreted as the average interference taken over time. Equation (1) with averaged interference power evaluated by load variables has been shown to give a good approximation for a multi-cell network especially at high data arrival rates [23]. Thus, equation (1) has been used in many applications [24], [26]–[29], as this formulation has a good structure with high accuracy for characterizing inter-cell interference. Note that the subcarrier assignment problem is always considered for the resource allocation problems without load vector [34]–[37]. Since the joint subcarrier assignment and power control problem involves integer variable, it is usually hard to solve.

Then, the achievable rate of user $j \in \mathcal{J}_i$ on subcarrier r can be written as

$$t_{ij}^r = m_{ij}^r B \log_2(1 + \eta_{ij}^r), \quad \forall i \in \mathcal{N}, j \in \mathcal{J}_i, r \in \mathcal{R}. \tag{2}$$

Denote $\boldsymbol{t}_i = [t_{i(J_{i-1}+1)}^1, \cdots, t_{iJ_i}^1, \cdots, t_{i(J_{i-1}+1)}^R, \cdots, t_{iJ_i}^R]$, and $\boldsymbol{t} = [\boldsymbol{t}_1, \cdots, \boldsymbol{t}_N]$. Note that the achievable rate t_{ij}^r in (2) can be regarded as a lower bound of the average achievable rate due to the convexity of $\log_2\left(1 + \frac{1}{x}\right)$.

Our aims are to minimize sum power, maximize sum rate and sum energy efficiency of all BSs, subject to the constraints of minimal rate for every user and maximal transmit power for each BS. From a mathematical optimization point of view, the unified formulation is given as follows

$$\min_{\boldsymbol{m}\geq\mathbf{0},\boldsymbol{p}\geq\mathbf{0},\boldsymbol{t}\geq\mathbf{0}} U(\boldsymbol{m},\boldsymbol{p},\boldsymbol{t}) \tag{3a}$$

$$\text{s.t.}\quad t_{ij}^r = m_{ij}^r B\log_2(1+\eta_{ij}^r),\quad \forall i,j,r \tag{3b}$$

$$\eta_{ij}^r = \frac{p_{ij}^r g_{ij}^r}{\sum_{k\in\mathcal{N}\setminus\{i\}}\sum_{l\in\mathcal{J}_k} m_{kl}^r p_{kl}^r g_{kj}^r + \sigma^2},\ \forall i,j,r \tag{3c}$$

$$\sum_{r\in\mathcal{R}} t_{ij}^r \geq d_{ij},\quad \forall i,j \tag{3d}$$

$$\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r \leq p_i^{\max},\quad \forall i \tag{3e}$$

$$\sum_{j\in\mathcal{J}_i} m_{ij}^r \leq 1,\quad \forall i,r, \tag{3f}$$

where $U(\boldsymbol{m},\boldsymbol{p},\boldsymbol{t})$ is the objective function, which can be sum power $\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r$, negative sum rate $-\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r$ or negative sum energy efficiency $-\sum_{i\in\mathcal{N}}\frac{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r}{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r}$. d_{ij} is the minimal rate of user $j\in\mathcal{J}_i$ served by BS i, and $p_i^{\max}$ denotes the maximal transmit power of BS i. Constraints (3b), (3c) and (3d) reflect that the minimal rate demand of each user should be satisfied. The sum transmit power of each BS should not exceed a maximal value, as stated in constraints (3e). Constraints (3f) represent the time division constraints.

The load coupling relation [30] is shown in constraints (3b) and (3c). Based on (3b) and (3c), the achievable rate t_{ij}^r of user $j\in\mathcal{J}_i$ on subcarrier r is determined by m_{ij}^r, p_{ij}^r, m_{kl}^r, and p_{kl}^r, $\forall k\in\mathcal{N}\setminus\{i\}$, $l\in\mathcal{J}_k$. As a result, once rate t_{ij}^r and power p_{ij}^r, p_{kl}^r, $\forall k\in\mathcal{N}\setminus\{i\}$, $l\in\mathcal{J}_k$ are given, the load m_{ij}^r of BS i is coupled with load m_{kl}^r of BS $k\in\mathcal{N}\setminus\{i\}$, which shows the load coupling relation.

Due to nonconvex constraints (3b), (3c) and (3e), Problem (3) is a nonconvex problem. It is difficult to obtain the globally optimal solution of a nonconvex problem even by the centralized algorithm. In the following, we devise three distributed algorithms to deal with the corresponding problems with low computational complexity.

III. Sum Power Minimization Problem

In this section, we investigate sum power minimization Problem (3) with

$$U(\boldsymbol{m},\boldsymbol{p},\boldsymbol{t}) = \sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r. \tag{4}$$

We first give the optimal conditions for load and rate vectors. Then, we provide a distributed power control and resource allocation algorithm. Finally, the detailed implementation and complexity analysis of the distributed algorithm are presented.

A. Optimal Conditions

We establish the optimal conditions for load vector $\boldsymbol{m}$ and rate vector $\boldsymbol{t}$.

Lemma 1: If Problem (3) is feasible, the optimal solution to minimize the total transmit power is such that the rate vector reaches the minimal rate constraints $\sum_{r\in\mathcal{R}} t_{ij}^r = d_{ij}$, $\forall i\in\mathcal{N}, j\in\mathcal{J}_i$, and load vector satisfies maximal load constraints $\sum_{j\in\mathcal{J}_i} m_{ij}^{r*} = 1$, $\forall i\in\mathcal{N}, r\in\mathcal{R}$.

Since Lemma 1 can be proved by using the same method in [31, Lemma 2], the proof of Lemma 1 is omitted. According to Lemma 1, transmitting with minimal rate is optimal in minimizing sum power, as less resources are used and hence less power is consumed. Moreover, we can also find that sum power minimization benefits from long transmit time from Lemma 1. The reason can be attributed to the proof of Lemma 2 in [31], which shows that as the transmit time increases, the required power decreases and then the product of time and power also decreases.

B. Distributed Algorithm

According to the definition of load variable, we introduce a set of new variables

$$q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r,\quad \forall i\in\mathcal{N}, \forall r\in\mathcal{R}, \tag{5}$$

which can be viewed as the total power of BS i on subcarrier r. Denote power vector $\boldsymbol{q}_i = [q_i^1,\cdots,q_i^R]$ and $\boldsymbol{q} = [\boldsymbol{q}_1,\cdots,\boldsymbol{q}_N]$. Substituting (1) and (5) into equation (2) yields

$$t_{ij}^r = m_{ij}^r B\log_2\left(1+\frac{p_{ij}^r g_{ij}^r}{\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2}\right). \tag{6}$$

Reformulating (6), we have

$$\begin{aligned} p_{ij}^r &= \frac{\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2}{g_{ij}^r}\left(\mathrm{e}^{\ln(2)t_{ij}^r/(Bm_{ij}^r)}-1\right)\\ &\triangleq f_{ij}^r(m_{ij}^r,\boldsymbol{q}_{-i}^r,t_{ij}^r),\quad \forall i\in\mathcal{N},\forall j\in\mathcal{J}_i,\forall r\in\mathcal{R}, \end{aligned} \tag{7}$$

where $\boldsymbol{q}_{-i}^r = [q_1^r,\cdots,q_{i-1}^r,q_{i+1}^r,\cdots,q_N^r]$.

From (7), we can observe that power vector $\boldsymbol{p}$ can be replaced by a new power vector $\boldsymbol{q}$ with fewer dimensions. With this observation, we have the following theorem.

Theorem 1: Sum power minimization Problem (3) with objective function (4) is equivalent to the following problem:

$$\min_{\boldsymbol{m}\geq\mathbf{0},\boldsymbol{q}\geq\mathbf{0},\boldsymbol{t}\geq\mathbf{0}} \sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}} q_i^r \tag{8a}$$

$$\text{s.t.}\quad \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r,\boldsymbol{q}_{-i}^r,t_{ij}^r) \leq q_i^r,\quad \forall i,r \tag{8b}$$

$$\sum_{r\in\mathcal{R}} t_{ij}^r \geq d_{ij},\quad \forall i,j \tag{8c}$$

$$\sum_{r\in\mathcal{R}} q_i^r \leq p_i^{\max},\quad \forall i \tag{8d}$$

$$\sum_{j\in\mathcal{J}_i} m_{ij}^r \leq 1,\quad \forall i,r. \tag{8e}$$

Proof: Please refer to Appendix A. □

According to Theorem 1, sum power minimization Problem (3) with objective function (4) is equivalent to Problem (8) with less variables. Besides, objective function (8a) and constraints (8c), (8d) and (8e) are all linear. However, Problem (8) is still noconvex due to nonconvex constraints (8b). To solve nonconvex Problem (8) in a distribute manner, the key idea of distributed algorithm is that each BS has the capability to design its own strategy until convergence [14], [38], [39].

Denoting $\boldsymbol{m}_{-i} = [\boldsymbol{m}_1, \cdots, \boldsymbol{m}_{i-1}, \boldsymbol{m}_{i+1}, \cdots, \boldsymbol{m}_N]$, $\boldsymbol{q}_{-i} = [\boldsymbol{q}_1, \cdots, \boldsymbol{q}_{i-1}, \boldsymbol{q}_{i+1}, \cdots, \boldsymbol{q}_N]$, and $\boldsymbol{t}_{-i} = [\boldsymbol{t}_1, \cdots, \boldsymbol{t}_{i-1}, \boldsymbol{t}_{i+1}, \cdots, \boldsymbol{t}_N]$, we can further obtain the following theorem.

Theorem 2: With load $\boldsymbol{m}_{-i}$, power $\boldsymbol{q}_{-i}$ and rate $\boldsymbol{t}_{-i}$ of other BSs fixed, the power minimization problem of BS i can be formulated as the following convex problem:

$$\min_{\boldsymbol{m}_i \geq \boldsymbol{0}, \boldsymbol{q}_i \geq \boldsymbol{0}, \boldsymbol{t}_i \geq \boldsymbol{0}} \quad \sum_{r \in \mathcal{R}} q_i^r \tag{9a}$$

$$\text{s.t.} \quad q_i^r \geq \sum_{j \in \mathcal{J}_i} a_{ij}^r m_{ij}^r (\mathrm{e}^{b t_{ij}^r / m_{ij}^r} - 1), \quad \forall r \tag{9b}$$

$$q_i^r \leq \bar{q}_i^r, \quad \forall r \tag{9c}$$

$$\sum_{j \in \mathcal{J}_i} m_{ij}^r \leq 1, \quad \forall r \tag{9d}$$

$$\sum_{r \in \mathcal{R}} t_{ij}^r \geq d_{ij}, \quad \forall j, \tag{9e}$$

where $a_{ij}^r = (\sum_{k \in \mathcal{N} \setminus \{i\}} q_k^r g_{kj}^r + \sigma^2)/g_{ij}^r$, $b = \ln(2)/B$ and $\bar{q}_i^r = \min_{k \in \mathcal{N} \setminus \{i\}} (q_k^r - \sum_{n \in \mathcal{N} \setminus \{k,i\}} u_{kn}^r q_n^r + v_k^r)/u_{ki}^r$ with u_{kn}^r, u_{ki}^r and v_k^r respectively defined in (21) and (22) in Appendix B, $\forall i \in \mathcal{N}$, $j \in \mathcal{J}_i$, $r \in \mathcal{R}$.

Proof: Please refer to Appendix B. □

To solve power minimization Problem (9), constraints (9b) hold with equality for the optimal solution, as otherwise the objective function (9a) can be further improved with satisfying all the constraints. Hence, the optimal transmit power of BS i on subcarrier r can be expressed as

$$q_i^r = \sum_{j \in \mathcal{J}_i} a_{ij}^r m_{ij}^r (\mathrm{e}^{b t_{ij}^r / m_{ij}^r} - 1). \tag{10}$$

Substituting (10) into Problem (9) yields the following equivalent problem

$$\min_{\boldsymbol{m}_i \geq \boldsymbol{0}, \boldsymbol{t}_i \geq \boldsymbol{0}} \quad \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}_i} a_{ij}^r m_{ij}^r (\mathrm{e}^{b t_{ij}^r / m_{ij}^r} - 1) \tag{11a}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{J}_i} a_{ij}^r m_{ij}^r (\mathrm{e}^{b t_{ij}^r / m_{ij}^r} - 1) \leq \bar{q}_i^r, \quad \forall r \tag{11b}$$

$$\sum_{j \in \mathcal{J}_i} m_{ij}^r \leq 1, \quad \forall r \tag{11c}$$

$$\sum_{r \in \mathcal{R}} t_{ij}^r \geq d_{ij}, \quad \forall j. \tag{11d}$$

According to Appendix B, Problem (11) can also be proved to be a convex problem, which can be effectively solved by standard convex optimization tools. In the following, we present our distributed power control and resource allocation for sum power minimization (DPCRA-SP) algorithm in Algorithm 1.

Note that for the sequential updating of Algorithm 1, each BS calculates its optimal load, power and rate vectors with the load, power and rate vectors of other BSs fixed. This guarantees the convergence of Algorithm 1, as discussed as follows.

Theorem 3: Assuming $N_{\max} \to \infty$, the sequence of load, power and rate vectors $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ generated by the sequential updating DPCRA-SP algorithm will converge.

Proof: Please refer to Appendix C. □

Algorithm 1 Distributed Power Control and Resource Allocation for Sum Power Minimization (DPCRA-SP)

1: Initialize any feasible $\boldsymbol{m}^{(0)} = [\boldsymbol{m}_1^{(0)}, \cdots, \boldsymbol{m}_N^{(0)}]$, $\boldsymbol{q}^{(0)} = [\boldsymbol{q}_1^{(0)}, \cdots, \boldsymbol{q}_N^{(0)}]$, $\boldsymbol{t}^{(0)} = [\boldsymbol{t}_1^{(0)}, \cdots, \boldsymbol{t}_N^{(0)}]$. Set the accuracy ϵ, the iteration number $n = 1$, and the maximal iteration number $N_{\max}$.
2: **for** $i = 1, 2, \cdots, N$ **do**
3: Let $\boldsymbol{m}_{-i}^{(n-1)} = [\boldsymbol{m}_1^{(n)}, \cdots, \boldsymbol{m}_{i-1}^{(n)}, \boldsymbol{m}_{i+1}^{(n-1)}, \cdots, \boldsymbol{m}_N^{(n-1)}]$, $\boldsymbol{q}_{-i}^{(n-1)} = [\boldsymbol{q}_1^{(n)}, \cdots, \boldsymbol{q}_{i-1}^{(n)}, \boldsymbol{q}_{i+1}^{(n-1)}, \cdots, \boldsymbol{q}_N^{(n-1)}]$, $\boldsymbol{t}_{-i}^{(n-1)} = [\boldsymbol{t}_1^{(n)}, \cdots, \boldsymbol{t}_{i-1}^{(n)}, \boldsymbol{t}_{i+1}^{(n-1)}, \cdots, \boldsymbol{t}_N^{(n-1)}]$, and $\boldsymbol{t}_i^* = \boldsymbol{t}_i^{(n-1)}$.
4: **repeat**
5: Obtain the optimal $\boldsymbol{m}_i^*$ and $\boldsymbol{t}_i^*$ of Problem (11) with fixed $\boldsymbol{t}_i^*$ $\boldsymbol{m}_{-i}^{(n-1)}$, $\boldsymbol{q}_{-i}^{(n-1)}$ and $\boldsymbol{t}_{-i}^{(n-1)}$.
6: **until** the objective function (11a) converges.
7: Set $\boldsymbol{m}_i^{(n)} = \boldsymbol{m}_i^*$, $\boldsymbol{t}_i^{(n)} = \boldsymbol{t}_i^*$ and obtain $\boldsymbol{q}_i^{(n)}$ from (10).
8: **end for**
9: If $n > N_{\max}$ or $\sum_{i \in \mathcal{N}} \sum_{r \in \mathcal{R}} |(q_i^r)^{(n-1)} - (q_i^r)^{(n)}| / \sum_{i \in \mathcal{N}} \sum_{r \in \mathcal{R}} (q_i^r)^{(n-1)} < \epsilon$, terminate. Otherwise, set $n = n + 1$ and go to step 2.

C. Implementation Method

To successfully implement the DPCRA-SP algorithm, BS i needs to compute load $\boldsymbol{m}_i$, power $\boldsymbol{q}_i$ and rate $\boldsymbol{t}_i$, which require the following information according to Problem (9): 1) coefficients $a_{ij}^r, \forall j \in \mathcal{J}_i, r \in \mathcal{R}$, 2) maximal transmit power $\bar{q}_i^r, \forall r \in \mathcal{R}$.

In order to obtain coefficients a_{ij}^r, user j should transmit the message of total received interference $I_j^r = \sum_{k \in \mathcal{N} \setminus \{i\}} q_k^r g_{kj}^r + \sigma^2$ to BS i. Then a_{ij}^r can be obtained as $a_{ij}^r = I_j^r / g_{ij}^r$, where channel gain g_{ij}^r can be estimated at BS i through the pilot sequence.

For maximal transmit power $\bar{q}_i^r$, each BS k transmits the message of power $q_{ki}^r \triangleq (q_k^r - \sum_{n \in \mathcal{N} \setminus \{k,i\}} u_{kn}^r q_n^r + v_k^r)/u_{ki}^r$ to BS i, $\forall k \in \mathcal{N} \setminus \{i\}, r \in \mathcal{R}$. To obtain q_{ki}^r, BS k needs four quantities: power q_n^r, power gain g_{nl}^r, g_{kl}^r and noise power σ^2, $\forall n \in \mathcal{N} \setminus \{k\}, r \in \mathcal{R}$. Since every BS broadcasts its power message to other BSs after updating transmit power, power q_n^r is always known by BS k. Power gain g_{nl}^r between user $l \in \mathcal{J}_k$ and BS n on subcarrier r is approximately estimated at BS k according to the location messages. Power gain g_{kl}^r can be estimated at BS k by the pilot sequence. It is assumed that the noise power σ^2 is always known at each BS.

Based on these quantities, each BS updates its load, power and rate vectors until the total interference power of each user converges.

D. Complexity analysis

For the simplicity of analysis, it is assumed that the number of users in each cell is M. For our proposed DPCRA-SP algorithm, the major complexity in each iteration lies in solving convex Problem (11), which almost involves a complexity of $\mathcal{O}(R^3 M^3)$ [40, Pages 487, 569]. As a result, the total complexity of the DPCRA-SP algorithm is $\mathcal{O}(K_{\text{SP}} R^3 M^3)$, where K_{SP} denotes the number of outer iterations of the DPCRA-SP algorithm.

For the optimal power vector for sum power minimization (OPV-SP) algorithm in [31], the main computational complexity lies in the bisection search of power, which involves a complexity of $\mathcal{O}(\log_2(1/\epsilon_2)RM)$ with accuracy ϵ_2. Hence, the total complexity of the OPV algorithm is $\mathcal{O}(K_{\text{OPV}}\log_2(1/\epsilon_2)NRM)$, where K_{OPV} denotes the total number of iterations of the OPV algorithm in [31].

IV. Sum Rate Maximization Problem

In this section, we investigate sum rate optimization Problem (3) with

$$U(\boldsymbol{m},\boldsymbol{p},\boldsymbol{t}) = -\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r. \tag{12}$$

To solve this problem, we provide a distributed power control and resource allocation algorithm along with the complexity analysis.

A. Distributed Algorithm

To simplify the original Problem (3) with objective function (12), we introduce vector $\boldsymbol{q}_i = [q_i^1, \cdots, q_i^R]$ and $\boldsymbol{q} = [\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N]$, where q_i^r is defined in (5), $\forall i \in \mathcal{N}, r \in \mathcal{R}$. According to the proof of Theorem 1, we can claim that sum rate maximization Problem (3) with objective function (12) is equivalent to the following problem:

$$\begin{aligned}
\min_{\boldsymbol{m}\geq\boldsymbol{0},\boldsymbol{q}\geq\boldsymbol{0},\boldsymbol{t}\geq\boldsymbol{0}} \quad & -\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r && \text{(13a)}\\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r) \leq q_i^r, \quad \forall i, r && \text{(13b)}\\
& \sum_{r\in\mathcal{R}} t_{ij}^r \geq d_{ij}, \quad \forall i, j && \text{(13c)}\\
& \sum_{r\in\mathcal{R}} q_i^r \leq p_i^{\max}, \quad \forall i && \text{(13d)}\\
& \sum_{j\in\mathcal{J}_i} m_{ij}^r \leq 1, \quad \forall i, r. && \text{(13e)}
\end{aligned}$$

Based on the proof of Theorem 2, the rate maximization problem with given load $\boldsymbol{m}_{-i}$, power $\boldsymbol{q}_{-i}$ and rate $\boldsymbol{t}_{-i}$ can be formulated as follows:

$$\begin{aligned}
\min_{\boldsymbol{m}_i\geq\boldsymbol{0},\boldsymbol{q}_i\geq\boldsymbol{0},\boldsymbol{t}_i\geq\boldsymbol{0}} \quad & -\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r && \text{(14a)}\\
\text{s.t.} \quad & q_i^r \geq \sum_{j\in\mathcal{J}_i} a_{ij}^r m_{ij}^r (\mathrm{e}^{bt_{ij}^r/m_{ij}^r} - 1), \quad \forall r && \text{(14b)}\\
& q_i^r \leq \bar{q}_i^r, \quad \forall r && \text{(14c)}\\
& \sum_{r\in\mathcal{R}} q_i^r \leq p_i^{\max} && \text{(14d)}\\
& \sum_{j\in\mathcal{J}_i} m_{ij}^r \leq 1, \quad \forall r && \text{(14e)}\\
& \sum_{r\in\mathcal{R}} t_{ij}^r \geq d_{ij}, \quad \forall j. && \text{(14f)}
\end{aligned}$$

According to Appendix B, Problem (14) is also a convex problem. Due to different objective functions between sum rate maximization Problem (14) and sum power minimization Problem (9), constraints (14b) do not hold with equality for sum rate maximization. To solve Problem (14), we use the interior-point method [40].

Through iteratively optimizing the load, power and rate vectors of one BS with the load, power and rate vectors of other BSs fixed, the distributed power control and resource allocation for sum rate maximization (DPCRA-SR) algorithm is presented in Algorithm 2.

Algorithm 2 Distributed Power Control and Resource Allocation for Sum Rate Maximization (DPCRA-SR)

1: Initialize any feasible $\boldsymbol{m}^{(0)} = [\boldsymbol{m}_1^{(0)}, \cdots, \boldsymbol{m}_N^{(0)}]$, $\boldsymbol{q}^{(0)} = [\boldsymbol{q}_1^{(0)}, \cdots, \boldsymbol{q}_N^{(0)}]$, $\boldsymbol{t}^{(0)} = [\boldsymbol{t}_1^{(0)}, \cdots, \boldsymbol{t}_N^{(0)}]$. Set the accuracy ϵ, the iteration number $n = 1$, and the maximal iteration number $N_{\max}$.
2: **for** $i = 1, 2, \cdots, N$ **do**
3: Let $\boldsymbol{m}_{-i}^{(n-1)} = [\boldsymbol{m}_1^{(n)}, \cdots, \boldsymbol{m}_{i-1}^{(n)}, \boldsymbol{m}_{i+1}^{(n-1)}, \cdots, \boldsymbol{m}_N^{(n-1)}]$, $\boldsymbol{q}_{-i}^{(n-1)} = [\boldsymbol{q}_1^{(n)}, \cdots, \boldsymbol{q}_{i-1}^{(n)}, \boldsymbol{q}_{i+1}^{(n-1)}, \cdots, \boldsymbol{q}_N^{(n-1)}]$ and $\boldsymbol{t}_{-i}^{(n-1)} = [\boldsymbol{t}_1^{(n)}, \cdots, \boldsymbol{t}_{i-1}^{(n)}, \boldsymbol{t}_{i+1}^{(n-1)}, \cdots, \boldsymbol{t}_N^{(n-1)}]$.
4: With $\boldsymbol{m}_{-i}^{(n-1)}$, $\boldsymbol{q}_{-i}^{(n-1)}$ and $\boldsymbol{t}_{-i}^{(n-1)}$ fixed, the optimal $\boldsymbol{m}_i^{(n)}$, $\boldsymbol{q}_i^{(n)}$ and $\boldsymbol{t}_i^{(n)}$ are obtained by solving convex Problem (14) with interior-point method.
5: **end for**
6: If $n > N_{\max}$ or $\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}|(t_{ij}^r)^{(n)} - (t_{ij}^r)^{(n-1)}|/\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}(t_{ij}^r)^{(n-1)} < \epsilon$, terminate. Otherwise, set $n = n + 1$ and go to step 2.

B. Complexity Analysis

For our proposed DPCRA-SR algorithm, the major complexity in each iteration lies in solving convex Problem (14). Considering that the dimension of the variables in Problem (14) is $2RM + R$, the complexity of solving Problem (14) by using the standard interior point method is $\mathcal{O}((2RM+R)^3) = \mathcal{O}(R^3M^3)$ [40, Pages 487, 569]. Denoting the total number of iterations of the DPCRA-SR algorithm by K_{SR}, the total complexity of the DPCRA-SR algorithm is $\mathcal{O}(K_{\text{SR}}NR^3M^3)$.

V. Sum Energy Efficiency Maximization Problem

In this section, we investigate sum energy efficiency optimization Problem (3) given by

$$U(\boldsymbol{m},\boldsymbol{p},\boldsymbol{t}) = -\sum_{i\in\mathcal{N}} \frac{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} t_{ij}^r}{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r}. \tag{15}$$

Note that equation (15) represents sum energy efficiency. The global energy efficiency is the ratio of the total amount of data that can be reliably transmitted per unit of time to the total amount of consumed power. In practice, however, the global energy efficiency may not be suitable to accurately reflect the energy efficiency performance of multi-cell networks since the BSs cannot share their power and have their own energy efficiency. Moreover, different BSs are equipped with different types of hardware and thus have different energy efficiency requirements. Under this consideration, the metric of sum energy efficiency recently attracts much attentions [14], [41]–[43]. On the other hand, the global energy efficiency is unable to control individual energy efficiency, which is important in multi-cell networks, and it only accounts for the energy efficiency of the entire network. Besides, sum energy efficiency provides more tuning abilities to control the energy efficiency of the individual BSs.

To solve sum energy efficiency optimization Problem (3), a distributed power control and resource allocation algorithm is first provided. Then, we present the complexity of this distributed algorithm.

A. Distributed Algorithm

To simplify Problem (3) with objective function (15), we introduce vector $\boldsymbol{q}_i=[q_i^1,\cdots,q_i^R]$ and $\boldsymbol{q}=[\boldsymbol{q}_1,\cdots,\boldsymbol{q}_N]$, where q_i^r is defined in (5), $\forall i\in\mathcal{N}, r\in\mathcal{R}$. Based on Theorem 1, we can also claim that sum energy efficiency maximization Problem (3) is equivalent to the following problem:

$$\min_{\boldsymbol{m}\geq\boldsymbol{0},\boldsymbol{q}\geq\boldsymbol{0},\boldsymbol{t}\geq\boldsymbol{0}} \quad -\sum_{i\in\mathcal{N}}\frac{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}t_{ij}^r}{\sum_{r\in\mathcal{R}}q_i^r} \tag{16a}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{J}_i}m_{ij}^r f_{ij}^r(m_{ij}^r,\boldsymbol{q}_{-i}^r,t_{ij}^r)\leq q_i^r,\quad \forall i,r \tag{16b}$$

$$\sum_{r\in\mathcal{R}}t_{ij}^r\geq d_{ij},\forall i,j \tag{16c}$$

$$\sum_{r\in\mathcal{R}}q_i^r\leq p_i^{\max},\quad\forall i \tag{16d}$$

$$\sum_{j\in\mathcal{J}_i}m_{ij}^r\leq 1,\quad\forall i,r. \tag{16e}$$

From Theorem 2, with load $\boldsymbol{m}_{-i}$, power $\boldsymbol{q}_{-i}$ and rate $\boldsymbol{t}_{-i}$ fixed, the energy efficiency maximization problem of BS i can be formulated as follows:

$$\min_{\boldsymbol{m}_i\geq\boldsymbol{0},\boldsymbol{q}_i\geq\boldsymbol{0},\boldsymbol{t}_i\geq\boldsymbol{0}} \quad -\frac{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}t_{ij}^r}{\sum_{r\in\mathcal{R}}q_i^r} \tag{17a}$$

$$\text{s.t.} \quad q_i^r\geq\sum_{j\in\mathcal{J}_i}a_{ij}^r m_{ij}^r(\mathrm{e}^{bt_{ij}^r/m_{ij}^r}-1),\quad\forall r \tag{17b}$$

$$q_i^r\leq\bar{q}_i^r,\quad\forall r \tag{17c}$$

$$\sum_{r\in\mathcal{R}}q_i^r\leq p_i^{\max} \tag{17d}$$

$$\sum_{j\in\mathcal{J}_i}m_{ij}^r\leq 1,\quad\forall r \tag{17e}$$

$$\sum_{r\in\mathcal{R}}t_{ij}^r\geq d_{ij},\quad\forall j. \tag{17f}$$

To solve sum energy efficiency optimization Problem (17), we transform the objective function (17a) with fractional form into an equivalent tractable form. Using the parametric approach in [44], we consider the following problem,

$$F_i(\zeta_i)=\min_{(\boldsymbol{m}_i,\boldsymbol{q}_i,\boldsymbol{t}_i)\in\mathcal{Q}}\zeta_i\sum_{r\in\mathcal{R}}q_i^r-\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}t_{ij}^r, \tag{18}$$

where the set $\mathcal{Q}$ is the feasible solution set of $(\boldsymbol{m}_i,\boldsymbol{q}_i,\boldsymbol{t}_i)$ satisfying constraints (17b)-(17f) and $\boldsymbol{m}_i\geq\boldsymbol{0},\boldsymbol{q}_i\geq\boldsymbol{0},\boldsymbol{t}_i\geq\boldsymbol{0}$. It is proved that solving (17) is equivalent to finding the root of the nonlinear function $F(\zeta)$ [45]. Thus, the energy efficiency maximization problem can be solve by using the Dinkelbach method as in [44], which is shown in Algorithm 3. Since Problem (18) with fixed ζ is a convex problem according to Appendix B, the optimal solution $(\boldsymbol{m}_i^*,\boldsymbol{q}_i^*,\boldsymbol{t}_i^*)$ in step 2 of Algorithm 3 can be effectively obtained by using the interior-point method.

Algorithm 3 The Dinkelbach Method

1: Initialize $\zeta_i=\zeta_i^{(0)}>0$. Set the accuracy ϵ and maximal iteration index $N_{\max}$, the iteration number $n=0$.
2: Use $\zeta_i=\zeta_i^{(n)}$ in (18) to obtain the optimal $(\boldsymbol{m}_i^*,\boldsymbol{q}_i^*,\boldsymbol{t}_i^*)$. Let $\zeta_i^{(n+1)}=\frac{\sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}(t_{ij}^r)^*}{\sum_{r\in\mathcal{R}}(q_i^r)^*}$.
3: If $n>N_{\max}$ or $|F_i(\zeta_i^{(n+1)})|<\epsilon$, terminate. Otherwise, set $n=n+1$ and go to step 2.

As a result, we propose a distributed power control and resource allocation for sum energy efficiency maximization (DPCRA-SEE) algorithm as in Algorithm 2, where step 4 is replaced by that the optimal $\boldsymbol{m}_i^{(n)}$, $\boldsymbol{q}_i^{(n)}$, and $\boldsymbol{t}_i^{(n)}$ are obtained via solving Problem (17) by using Algorithm 3.

B. Complexity Analysis

For the proposed DPCRA-SEE algorithm, in each iteration the complexity lies in solving convex Problem (17), which almost involves a complexity of $\mathcal{O}(R^3M^3)$ [40, Pages 487, 569]. Hence, the total complexity of the DPCRA-SEE algorithm is $\mathcal{O}(K_{\text{SEE}}K_{\text{D}}NR^3M^3)$, where K_{SEE} denotes the total number of the iterations of the DPCRA-SEE algorithm, and K_{D} is the total number of the iterations by using the Dinkelbach method.

VI. Numerical Results

In this section, we evaluate the performance of the proposed algorithms through numerical results. As shown in Fig. 2, the three-site 3GPP LTE network layout is adopted and the total number of cells is 15. There are a total number of 450 users (blue dots and red dots in Fig. 2). One half users are generated with one hotspot of 70 m radius per macro cell area, while the other half users are uniformly distributed in the whole area. The total number of subcarriers is 128 for each cell. Besides, the bandwidth of each subcarrier is the same and the total bandwidth of the network is 18 MHz.

In modeling the propagation environment, we use the large-scale path loss $L(d)=128.1+37.6\log(d)$, d is in km, and the small scale fading is modeled as Rayleigh fading with unit variance. The red dots in Fig. 2 represent users belonging to cell 9. We assume equal maximal transmit power (i.e., $p_i^{\max}=10$ W, $\forall i\in\mathcal{N}$) for all BSs and equal rate demands (i.e., $d_{ij}=D$, $\forall i\in\mathcal{N}$, $j\in\mathcal{J}_i$) for all users. We compare the proposed distributed algorithms with the OPV-SP algorithm in [31], where the transmit power for users in the same cell is the same.

The sum power versus different rate demands for a multi-cell network is shown in Fig. 3. According to Fig. 3, DPCRA-SP outperforms the other three algorithms and sum power is greatly reduced by using DPCRA-SP compared to OPV-SP when the rate demand is large. According to the complexity

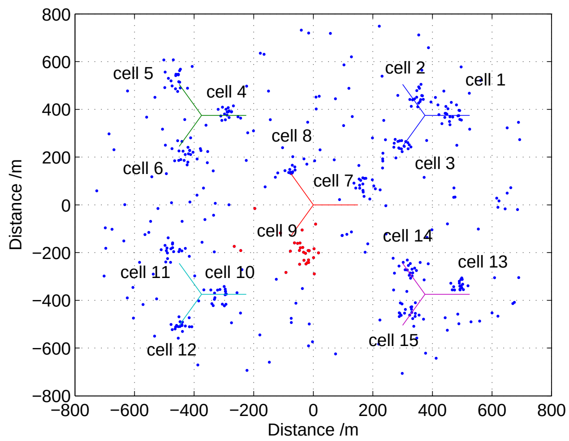

Fig. 2. Network configuration and user distribution.

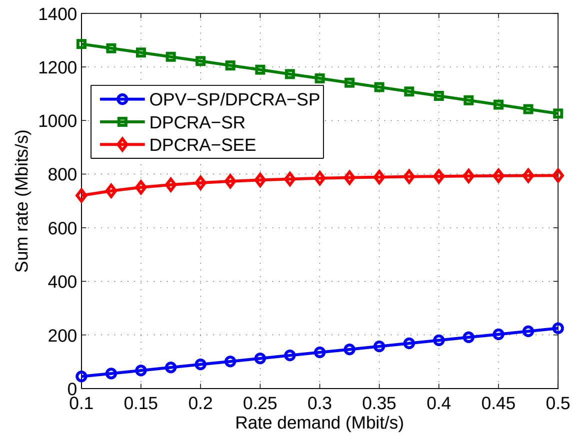

Fig. 4. Sum rate versus minimal rate demand of each user.

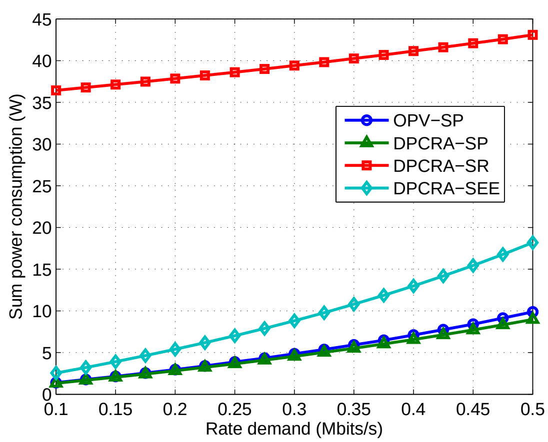

Fig. 3. Sum power versus minimal rate demand of each user.

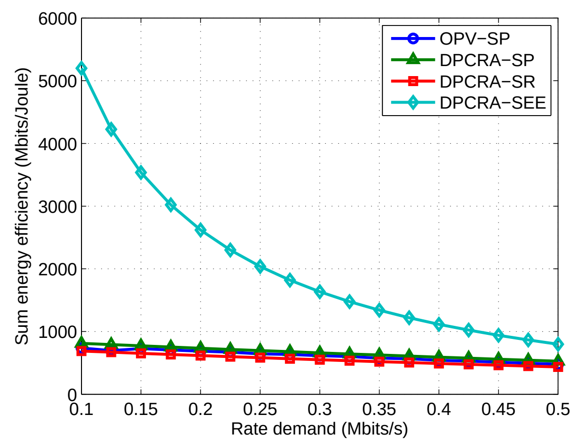

Fig. 5. Sum energy efficiency versus minimal rate demand of each user.

analysis in Section III-D, the proposed DPCRA-SP requires more computations than OPV-SP. Thus, we can conclude that the proposed DPCRA-SP achieves a significant performance gain at the cost of some additional computations. It is also observed that sum power monotonically increases with the minimal rate demand for all the four algorithms. This is because large transmit power should be allocated to the users to meet the high minimal rate demand.

Fig. 4 illustrates the sum rate versus different rate demands for a multi-cell network. From Fig. 4, the sum rate of DPCRA-SR is the largest among four algorithms. Besides, sum rate monotonically decreases with the minimal rate demand for DPCRA-SR. This is due to the fact that high rate demand requires large transmit power of each BS, causing large mutual interference among cells and low achievable rate for users. With the increase of the rate demand of the users, the sum rate of DPCRA-SEE almost remains the same. Besides, the sum rate of DPCRA-SP or OPV-SP increases with the rate demand of each user. This can be explained by Lemma 1, from which the achievable rate of each user should be equal to the minimal rate demand for energy minimization.

The sum energy efficiency versus different rate demands for a multi-cell network is presented in Fig. 5. From Fig. 5, it is observed that the energy efficiency of DPCRA-SEE is superior over the other three algorithms especially when the rate demand is low. The reason is that the other three algorithms aim at optimizing other performance criteria, rather than the energy efficiency that is measured in Mbits/Joule. However, the DPCRA-SEE directly optimizes the sum energy efficiency. It is also observed that the energy efficiency of DPCRA-SEE decreases with the minimal rate demand. This is because that high rate demand requires large transmit power of each BS, which overwhelms the increase of data rate, leading to the decrease of sum energy efficiency. Based on Fig. 3 to Fig. 5, we find that DPCRA-SP, DPCRA-SR and DPCRA-SEE respectively achieve the best sum power, sum rate and sum energy efficiency, which show the effectiveness of the proposed distributed algorithms.

VII. Conclusion

In this paper, we studied sum power minimization, sum rate maximization and sum energy efficiency maximization problems with load coupling for multi-cell OFDM networks over frequency selective fading channels. To solve each problem, we proposed a low-complexity distributed power control and resource allocation algorithm. Through simulations, the proposed sum power minimization algorithm achieves better performance than conventional sum power minimization algorithm at the cost of some additional computations. Besides, due to mutual interference, the sum rate of the proposed sum rate maximization algorithm decreases with the rate demand. Using the proposed sum energy efficiency maximization algorithm, the energy efficiency performance can be significantly improved especially at low rate demand.

Appendix A
Proof of theorem 1

To show the equivalence, we note that if the pair $(\boldsymbol{m}, \boldsymbol{p}, \boldsymbol{t})$ is feasible in (3), then the pair $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$, where power $q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r$, $\forall i \in \mathcal{N}$, is feasible in (8), with the same objective value $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r = \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r$. It follows that the optimal value of (3) is greater than or equal to the optimal value of (8).

Conversely, if $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ is the optimal solution to Problem (8), we can claim that

$$q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r), \quad \forall i \in \mathcal{N}, r\in\mathcal{R}.$$

If there exists at least one q_i^r which satisfies

$$q_i^r > \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r).$$

Let

$$\tilde{q}_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r) < q_i^r. \tag{19}$$

Denote $\tilde{\boldsymbol{q}}_i = [q_i^1, \cdots, q_i^{r-1}, \tilde{q}_i^r, q_i^{r+1}, \cdots, q_i^R]$, and $\tilde{\boldsymbol{q}} = [\boldsymbol{q}_1, \cdots, \boldsymbol{q}_{i-1}, \tilde{\boldsymbol{q}}_i, \boldsymbol{q}_{i+1}, \cdots, \boldsymbol{q}_N]$. According to (7) and (8b), we have

$$\begin{aligned}
&\sum_{l\in\mathcal{J}_k} m_{kl}^r f_{kl}^r(m_{kl}^r, \tilde{\boldsymbol{q}}_{-k}^r, t_{kl}^r)\\
&= \sum_{l\in\mathcal{J}_k} m_{kl}^r \frac{\sum_{n\in\mathcal{N}\setminus\{i,k\}} q_n^r g_{nl}^r + \sigma^2}{g_{kl}^r}(\mathrm{e}^{\ln(2)t_{kl}^r/(Bm_{kl}^r)}-1)\\
&\quad + \sum_{l\in\mathcal{J}_k} m_{kl}^r \frac{\tilde{q}_i^r g_{il}^r}{g_{kl}^r}(\mathrm{e}^{\ln(2)t_{kl}^r/(Bm_{kl}^r)}-1)\\
&< \sum_{l\in\mathcal{J}_k} m_{kl}^r \frac{\sum_{n\in\mathcal{N}\setminus\{i,k\}} q_n^r g_{nl}^r + \sigma^2}{g_{kl}^r}(\mathrm{e}^{\ln(2)t_{kl}^r/(Bm_{kl}^r)}-1)\\
&\quad + \sum_{l\in\mathcal{J}_k} m_{kl}^r \frac{q_i^r g_{il}^r}{g_{kl}^r}(\mathrm{e}^{\ln(2)t_{kl}^r/(Bm_{kl}^r)}-1)\\
&= \sum_{l\in\mathcal{J}_k} m_{kl}^r f_{kl}^r(m_{kl}^r, \boldsymbol{q}_{-k}^r, t_{kl}^r) \le q_k^r, \quad \forall k \neq i.
\end{aligned} \tag{20}$$

Therefore, $(\boldsymbol{m}, \tilde{\boldsymbol{q}}, \boldsymbol{t})$ is feasible with $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} \tilde{q}_i^r < \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r$, which contradicts the fact that $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ is the optimal solution. Thus, $q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r)$, $\forall i \in \mathcal{N}, r\in\mathcal{R}$. The pair $(\boldsymbol{m}, \boldsymbol{p}, \boldsymbol{t})$, where $p_{ij}^r = f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r)$, $\forall i\in\mathcal{N}, j\in\mathcal{J}_i, r\in\mathcal{R}$, is feasible in Problem (3) with the same objective value $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r = \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r$. As a result, we conclude that the optimal value of (3) is less than or equal to the optimal value of (8). Hence, Problem (3) is equivalent to Problem (8).

Appendix B
Proof of Theorem 2

We first show Problem (8) with given load $\boldsymbol{m}_{-i}$ power $\boldsymbol{q}_{-i}$, and rate $\boldsymbol{t}_{-i}$ can be formulated as (9). In order to rewrite constraints (8b) in a convenient form, we let

$$u_{kn}^r = \sum_{l\in\mathcal{J}_k} \frac{g_{nl}^r\left(\mathrm{e}^{\frac{\ln(2)t_{kl}^r}{Bm_{kl}^r}}-1\right)}{g_{kl}^r}, \quad \forall k,n\in\mathcal{N}, k\neq n, r\in\mathcal{R}, \tag{21}$$

and

$$v_k^r = \sum_{l\in\mathcal{J}_k} \sigma^2/g_{kl}^r, \quad \forall k,n\in\mathcal{N}, k\neq n, r\in\mathcal{R}. \tag{22}$$

According to (21) and (22), inequality constraints (8b) can be reformulated as

$$q_k^r \ge \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn}^r q_n^r + u_{ki}^r q_i^r - v_k^r, \quad \forall k\in\mathcal{N}\setminus\{i\}, \tag{23}$$

and

$$q_i^r \ge \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}^r, t_{ij}^r). \tag{24}$$

With given power $\boldsymbol{q}_{-i}$, load $\boldsymbol{m}_{-i}$ and rate $\boldsymbol{t}_{-i}$, inequality constraints (23) are equivalent to

$$q_i^r \le \frac{\left(q_k^r - \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn}^r q_n^r + v_k^r\right)}{u_{ki}^r}, \ \forall k\in\mathcal{N}\setminus\{i\}. \tag{25}$$

Note that constraints (8d) can be omitted, since the optimal solution to sum power minimization Problem (9) always satisfies constraints (8d). Hence, Problem (8) with given load $\boldsymbol{m}_{-i}$, power $\boldsymbol{q}_{-i}$ and rate $\boldsymbol{t}_{-i}$ can be formulated as Problem (9).

Then, we show that Problem (9) is convex. Since objective function (9a) and constraints (9c), (9d) and (9e) are all linear, we only need to check that constraints (9b) are convex. According to [40, Page 89], the perspective of $w(x)$ is the function $z(x,t)$ defined by $z(x,t) = tw(x/t)$, $\textbf{dom}\ z = \{(x,t)|x/t \in \textbf{dom}\ w, t>0\}$. If $w(x)$ is a convex function, then so is its perspective function $z(x,t)$ [40, Page 89]. Since $w(t_{ij}^r) = a_{ij}^r(\mathrm{e}^{bt_{ij}^r}-1)$ is convex with respect to (w.r.t.) t_{ij}^r, the perspective function $z(t_{ij}^r, m_{ij}^r) = a_{ij}^r m_{ij}^r(\mathrm{e}^{bt_{ij}^r/m_{ij}^r}-1)$ is convex w.r.t. (t_{ij}^r, m_{ij}^r). Owing to that $\sum_{j\in\mathcal{J}_i} a_{ij}^r m_{ij}^r(\mathrm{e}^{bt_{ij}^r/m_{ij}^r}-1)$ is a nonnegative weighted sum of convex functions, we can find that constraints (9b) are convex w.r.t. $(\boldsymbol{m}_i, \boldsymbol{q}_i, \boldsymbol{t}_i)$ [40, Page 79].

Appendix C
Proof of Theorem 3

The proof is established by showing that when one BS updates its load, power and rate vectors by solving Problem (9), the sum power of all BSs is non-increasing. Denote $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ as the feasible solution to Problem (8) before BS i starts to update its load, power and rate vectors. Let $(\tilde{\boldsymbol{m}}_i, \tilde{\boldsymbol{q}}_i, \tilde{\boldsymbol{t}}_i)$ denote the updated load, power and rate vectors of BS i with given $(\boldsymbol{m}_{-i}, \boldsymbol{q}_{-i}, \boldsymbol{t}_{-i})$. We further set

$$\tilde{\boldsymbol{m}} = [\boldsymbol{m}_1, \cdots, \boldsymbol{m}_{i-1}, \tilde{\boldsymbol{m}}_i, \boldsymbol{m}_{i+1}, \cdots, \boldsymbol{m}_N], \tag{26}$$

$$\tilde{\boldsymbol{q}} = [\boldsymbol{q}_1, \cdots, \boldsymbol{q}_{i-1}, \tilde{\boldsymbol{q}}_i, \boldsymbol{q}_{i+1}, \cdots, \boldsymbol{q}_N], \tag{27}$$

and

$$\tilde{\boldsymbol{t}} = [\boldsymbol{t}_1, \cdots, \boldsymbol{t}_{i-1}, \tilde{\boldsymbol{t}}_i, \boldsymbol{t}_{i+1}, \cdots, \boldsymbol{t}_N]. \tag{28}$$

From (9b)-(9e), it can be obtained that $(\tilde{\boldsymbol{m}}, \tilde{\boldsymbol{q}}, \tilde{\boldsymbol{t}})$ is also a feasible solution to Problem (8). Then, we have

$$\begin{aligned} \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}} q_k^r &= \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r + \sum_{r\in\mathcal{R}} q_i^r \\ &\geq \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r + \sum_{r\in\mathcal{R}} \tilde{q}_i^r = \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}} \tilde{q}_k^r, \end{aligned} \tag{29}$$

where the inequality follows from the fact that $(\tilde{\boldsymbol{m}}_i, \tilde{\boldsymbol{q}}_i, \tilde{\boldsymbol{t}}_i)$ is the optimal strategy of BS i for power minimization by solving Problem (8) with given $(\boldsymbol{m}_{-i}, \boldsymbol{q}_{-i}, \boldsymbol{t}_{-i})$. Since the sum power (8a) is nonincreasing in each iteration according to (29) and the sum power (8a) is finitely lower-bounded (i.e., nonnegative), the DCPRA-SP algorithm must converge.

References

[1] Z. Yang, H. Xu, J. Shi, Y. Pan, and Y. Li, "Power control and resource allocation for multi-cell OFDM networks," in *IEEE Int. Conf. Compt. Commun. Workshops*, San Francisco, CA, USA, Apr. 2016, pp. 891–896.

[2] V. W. Wong, R. Schober, D. W. K. Ng, and L.-C. Wang, *Key Technologies for 5G Wireless Systems*. Cambridge University Press, 2017.

[3] C. Y. Wong, R. S. Cheng, K. B. Lataief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.

[4] D. Kivanc, G. Li, and H. Liu, "Computationally efficient bandwidth allocation and power control for OFDMA," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1150–1158, Nov. 2003.

[5] C. Pan, J. Wang, W. Zhang, B. Du, and M. Chen, "Power minimization in multi-band multi-antenna cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 5056–5069, Sep. 2014.

[6] E. Boshkovska, D. W. K. Ng, N. Zlatanov, and R. Schober, "Practical non-linear energy harvesting model and resource allocation for SWIPT systems," *IEEE Commun. Lett.*, vol. 19, no. 12, pp. 2082–2085, 2015.

[7] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.

[8] W. Xu, C. Zhao, P. Zhou, and Y. Yang, "Efficient adaptive resource allocation for multiuser OFDM systems with minimum rate constraints," in *Proc. IEEE Int. Conf. Commun.*, Glasgow, UK, Jun. 2007, pp. 5126–5131.

[9] S. Corroy, L. Falconetti, and R. Mathar, "Dynamic cell association for downlink sum rate maximization in multi-cell heterogeneous networks," in *Proc. IEEE Int. Conf. Commun.*, Ottawa, ON, Canada, Jun. 2012, pp. 2457–2461.

[10] L. Venturino, A. Zappone, C. Risi, and S. Buzzi, "Energy-efficient scheduling and power allocation in downlink OFDMA networks with base station coordination," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 1–14, Jan. 2015.

[11] B. Dai and W. Yu, "Energy efficiency of downlink transmission strategies for cloud radio access networks," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 1037–1050, Apr. 2016.

[12] C. Xiong, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient resource allocation in OFDMA networks," *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3767–3778, Dec. 2012.

[13] F. Meshkati, H. V. Poor, S. C. Schwartz, and R. V. Balan, "Energy-efficient resource allocation in wireless networks with quality-of-service constraints," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3406–3414, Nov. 2009.

[14] C. Pan, W. Xu, J. Wang, H. Ren, W. Zhang, N. Huang, and M. Chen, "Pricing-based distributed energy-efficient beamforming for MISO interference channels," *IEEE J. Sel. Areas Commu.*, vol. 34, no. 4, pp. 710–722, Apr. 2016.

[15] A. Ouni, H. Rivano, F. Valois, and C. Rosenberg, "Energy and throughput optimization of wireless mesh networks with continuous power control," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 1131–1142, Feb. 2015.

[16] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," *Found. Trends Netw.*, vol. 1, no. 1, pp. 1–144, 2006.

[17] Z. Yang, C. Pan, W. Xu, H. Xu, and M. Chen, "Joint time allocation and power control in multi-cell networks with load coupling: Energy saving and rate improvement," *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1–1, 2017.

[18] K. Seong, M. Mohseni, and J. M. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *Proc. IEEE Int. Symp. Inf. Theory*, Seattle, WA, USA, Dec. 2006, pp. 1394–1398.

[19] D. López-Pérez, X. Chu, A. V. Vasilakos, and H. Claussen, "Power minimization based resource allocation for interference mitigation in OFDMA femtocell networks," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 2, pp. 333–344, 2014.

[20] S. Wang, Z.-H. Zhou, M. Ge, and C. Wang, "Resource allocation for heterogeneous multiuser OFDM-based cognitive radio networks with imperfect spectrum sensing," in *Proc. IEEE Int. Conf. Compt. Commun*, Orlando, FL, USA, May 2012, pp. 2264–2272.

[21] Y. Hua, Q. Zhang, and Z. Niu, "Resource allocation in multi-cell OFDMA-based relay networks," in *Proc. IEEE Int. Conf. Compt. Commun*, San Diego, CA, USA, May 2010, pp. 1–9.

[22] G. Zhang, K. Yang, J. Wei, K. Xu, and P. Liu, "Virtual resource allocation for wireless virtualization networks using market equilibrium theory," in *Proc. IEEE Int. Conf. Compt. Commun Workshops*, Hong Kong, China, Aug. 2015, pp. 366–371.

[23] A. J. Fehske and G. P. Fettweis, "Aggregation of variables in load models for interference-coupled cellular data networks," in *Proc. IEEE Int. Conf. Commun.* Ottawa, ON, Canada: IEEE, Nov. 2012, pp. 5102–5107.

[24] I. Siomina and D. Yuan, "On constrained cell load coupling with applications in LTE networks," *IEEE Commun. Lett.*, vol. 18, no. 10, pp. 1771–1774, Oct. 2014.

[25] Z. Yang, J. Shi, H. Xu, Y. Pan, and M. Chen, "Energy-efficient optimization with cell load coupling for ofdm networks," in *Proc. IEEE Veh. Technol. Conf.*, Nanjing, China, May. 2016, pp. 1–5.

[26] I. Siomina and D. Yuan, "Load balancing in heterogeneous LTE: Range optimization via cell offset and load-coupling characterization," in *Proc. IEEE Int. Conf. Commun.*, Ottawa, ON, Canada, Jun. 2012, pp. 1357–1361.

[27] ——, "Optimization approaches for planning small cell locations in load-coupled heterogeneous LTE networks," in *Proc. IEEE Annu. Symp. Personal, Indoor and Mobile Radio Commun.*, London, UK, Sep. 2013, pp. 2904–2908.

[28] C. K. Ho, D. Yuan, and S. Sun, "Data offloading in load coupled networks: A utility maximization framework," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 1921–1931, Apr. 2014.

[29] L. You and D. Yuan, "Load optimization with user association in cooperative and load-coupled LTE networks," *IEEE Trans. Wireless Commun.*, vol. PP, no. 99, pp. 1–1, Mar. 2017.

[30] I. Siomina and D. Yuan, "Analysis of cell load coupling for LTE network planning and optimization," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2287–2297, Jun. 2012.

[31] C. K. Ho, D. Yuan, L. Lei, and S. Sun, "Power and load coupling in cellular networks for energy optimization," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 509–519, Aug. 2015.

[32] I. Siomina, A. Furuskar, and G. Fodor, "A mathematical framework for statistical QoS and capacity studies in OFDM networks," in *Proc. IEEE Annu. Symp. Personal, Indoor and Mobile Radio Commun.* Tokyo, Japan: IEEE, Apr. 2009, pp. 2772–2776.

[33] K. Majewski and M. Koonert, "Conservative cell load approximation for radio networks with shannon channels and its application to LTE network planning," in *Advanced Int. Conf. Telecommun.* Barcelona, Spain: IEEE, Jun. 2010, pp. 219–225.

[34] M. Pischella and J. C. Belfiore, "Distributed resource allocation for rate-constrained users in multi-cell OFDMA networks," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 250–252, Apr. 2008.

[35] G. Li and H. Liu, "Downlink radio resource allocation for multi-cell OFDMA system," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3451–3459, Dec. 2006.

[36] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation in multi-cell OFDMA systems with limited backhaul capacity," *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3618–3631, Oct. 2012.

[37] Z. Han, Z. Ji, and K. J. R. Liu, "Non-cooperative resource competition game by virtual referee in multi-cell OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, pp. 1079–1090, Aug. 2007.

[38] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Prentice hall Englewood Cliffs, NJ, 1989, vol. 23.

[39] C. Pan, W. Xu, J. Wang, H. Ren, W. Zhang, N. Huang, and M. Chen, "Totally distributed energy-efficient transmission in MIMO interference channels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 11, pp. 6325–6338, Nov. 2015.

[40] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[41] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Foundations and Trends in Communications and Information Theory*, vol. 11, no. 3-4, pp. 185–396, 2015.

[42] W. Xu, Y. Cui, H. Zhang, G. Y. Li, and X. You, "Robust beamforming with partial channel state information for energy efficient networks," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 12, Dec. 2015.

[43] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Transactions on Wireless Communications*, vol. 11, no. 8, pp. 2946–2957, Aug. 2012.

[44] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, 1967.

[45] F. Haider, C.-X. Wang, H. Haas, E. Hepsaydir, and X. Ge, "Energy-efficient subcarrier-and-bit allocation in multi-user OFDMA systems," in *Proc. IEEE Veh. Technol. Conf.*, Yokohama, Japan, Jul. 2012, pp. 1–5.